Nonlinear Forced Vibration of Orthotropic Rectangular Plates Using the Method of Multiple Scales

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The method of multiple scales (MMS) in conjunction with the Galerkin method is used to analyze the nonlinear forced and damped response of a rectangular orthotropic plate subjected to a uniformly distributed harmonic transverse loading. The effects of damping and in-plane loads are considered. The analysis considers simply supported as well as clamped plates. For each case, both movable and immovable edge conditions are considered. By using MMS, all possible resonances such as primary, subharmonic, and superharmonic resonances are studied. For the undamped response without in-plane loading, comparisons of the MMS results with those obtained by the finite-element method show excellent agreement.

Introduction

DEVELOPMENT of composite materials comprising laminates of orthotropic or multilayered anisotropic materials recently has been receiving substantially growing research efforts. Due to the increasing demands for energy-efficient, high strength, minimum weight aircraft designs, many researchers believe that the use of composite materials offers promising alternatives for aircraft designs. Thin, laminated, composite plates subjected to transverse periodic loadings could encounter deflections of the order of plate thickness or even higher. Responses of this kind cannot be predicted by using the linear theory. Consequently, the need to study large-amplitude-deflection vibrations of composite structures is of paramount importance.

The literature survey shows that the equations of motion for the large deflection analysis of heterogeneous anisotropic plates using the von Kármán geometrical nonlinearity were first considered by Whitney and Leissa.[1] Based on these equations, different methods of analysis have been developed by several researchers. An excellent number of collections on nonlinear free and forced vibrations of composite plates covering the work through 1979 can be found in the comprehensive book by Chia.[2] Bert[3] has conducted a survey on the dynamics of composite plates for the period 1979–81. A review of the literature on nonlinear vibrations of plates can be found in the review paper by Sathyamoorthy[4] and the book by Nayfeh and Mook.[5]

Large deflection analysis of symmetrically laminated rectangular plates subjected to transverse harmonic loading is studied by Gray et al.[6] using the Galerkin method. Mei and Chiang[7] used a finite-element method to analyze the nonlinear forced vibration of symmetrically laminated rectangular plates. Later, Wentz et al.[8] applied the finite-element method to study the forced vibration response of generally laminated rectangular plates. To the best of our knowledge, the effects of damping and in-plane loadings in the large-deflection vibration of orthotropic plates have been neglected in the literature.

The purpose of the present paper is to study the nonlinear vibrations of rectangular orthotropic plates including the effects of damping and initial in-plane loadings. The governing equations of motion are presented in terms of the lateral displacement and stress function. The equations are nondimensionalized following the transformation introduced by Brunelle and Oyibo.[9] Though multimode analysis can be treated, the present study is focused on single-mode analysis. A deflection function representing the first mode and satisfying the boundary conditions is assumed, and subsequently the stress function is found. Next, the Galerkin method is applied to obtain the modal equation, which is solved analytically by using the method of multiple scales (MMS).[10] The MMS also provides solutions for subharmonic and superharmonic resonances. The effects of damping ratio, plate aspect ratio, and in-plane loading then are studied.

Formulation

Governing Equations

Using the von Kármán large-deflection analysis and the classical stress-strain relations for a rectangular orthotropic plate,[6,11] including the damping force and the applied in-plane loads, the following differential equations are obtained

$$L(W,\Phi) = \rho h\ddot{W} + g\dot{W} + D_{11}W,_{xxxx} + 2(D_{12} + 2D_{66})W,_{xxyy} + D_{22}W,_{yyyy} - [(\Phi,_{yy} + N_x^a)W,_{xx} + (\Phi,_{xx} + N_y^a)W,_{yy} - 2\Phi,_{xy}W,_{xy}] - P(t) = 0 \tag{1}$$

$$A_{22}^*\,\Phi,_{xxxx} + (2A_{12}^* + A_{66}^*)\,\Phi,_{xxyy} + A_{11}^*\,\Phi,_{yyyy} = W,_{xy}^2 - W,_{xx}W,_{yy} \tag{2}$$

where W is the panel deflection, ρ the density, h the plate thickness, N_x^a and N_y^a the applied in-plane loads, Φ the Airy stress function, A_{ij}^* and D_{ij} the extensional and bending stiffnesses, and $P(t)$ the external excitations. For a uniformly distributed loading, $P(t)$ is assumed to be $P_0 \cos\omega t$. To solve Eqs. (1) and (2), a complete set of boundary conditions must be specified. The cases of boundary conditions considered here are either all edges are simply supported or all edges are clamped. The edge conditions considered are for both movable and immovable edges. For movable edges, the in-plane boundary conditions are

at $x = 0,\ a$:

$$\Phi,_{xy} = 0,\quad \int_0^b \Phi,_{yy}\,dy = 0 \tag{3a}$$

at $y = 0,\ b$:

$$\Phi,_{xy} = 0,\quad \int_0^a \Phi,_{xx}\,dx = 0 \tag{3b}$$

Received Feb. 18, 1987; presented as Paper 87-0855-CP at the AIAA/ASME/AHS/ASEE 28th Structure, Structural Dynamics and Materials Conference, Monterey, CA, April 6–8, 1987; revision received July 25, 1988.

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where a and b are the plate dimensions. For immovable edges, the in-plane boundary conditions are

at $x = 0, a$:

$$\Phi,_{xx} = 0, \quad \int_0^b \int_0^a \left(\epsilon_x^0 - \frac{1}{2} w,_x^2\right) dx\, dy = 0 \tag{4a}$$

at $y = 0, b$:

$$\Phi,_{xy} = 0, \quad \int_0^b \int_0^a \left(\epsilon_y^0 - \frac{1}{2} w,_y^2\right) dx\, dy = 0 \tag{4b}$$

Introducing the following transformation for the variables

$$x = \left(\frac{D_{11}}{D_{22}}\right)^{1/4} a_0 \xi, \quad y = b_0 \eta, \quad \tau = t_1 \left(\frac{D_{22}}{\rho h a_0^4}\right)^{1/2}$$

$$F = \Phi/(D_{11}\, D_{22})^{1/2}, \quad w = W/h \tag{5}$$

where $a_0 = (D_{22}/D_{11})^{1/4} a$ and $b_0 = b$, Eqs. (1) and (2) are transformed to the following nondimensional form

$$L(w,F) = w,_{\tau\tau} + C w,_\tau + 2D^* r_0^2 w,_{\xi\xi\eta\eta} + r_0^4 w,_{\eta\eta\eta\eta}$$

$$- r_0^2[(F,_{\eta\eta} + N_\xi^a)\, w,_{\xi\xi} + (F,_{\xi\xi} + N_\eta^a)\, w,_{\eta\eta}$$

$$- 2\, F,_{\xi\eta} w,_{\xi\eta}] - P_1(\tau) = 0 \tag{6}$$

$$\alpha\, F,_{\xi\xi\xi\xi} + \beta r^2 F,_{\xi\xi\eta\eta} + \gamma r^4 F,_{\eta\eta\eta\eta} = r^2\, (w^2,_{\xi\eta} - w,_{\xi\xi} w,_{\eta\eta}) \tag{7}$$

where

$$r = a/b, \quad r_0 = a_0/b_0, \quad C = g a^2/(\rho h D_{22})^{1/2} \tag{8}$$

Method of Solution

The Galerkin method is employed to transform Eqs. (6) and (7) into an ordinary, nonlinear differential equation. The deflection function w for a single-mode solution is chosen as

$$w = q(\tau)\, \phi\, (\xi,\eta) \tag{9}$$

in which $\phi(\xi,\eta)$ is the admissible function that satisfies the boundary conditions. Equation (9) is substituted into Eq. (7) and the solution for the resulting linear partial differential equation in terms of F can be written as

$$F(\xi,\eta,\tau) = F^h\, (\xi,\eta,\tau) + F^P\, (\xi,\eta,\tau) \tag{10}$$

where F^h is the homogeneous solution that includes the contribution from the in-plane stresses independent of deflection, and F^p is the particular solution that includes the contribution from the out-of-plane boundary conditions. The homogeneous solution is determined by using the in-plane boundary conditions given by Eqs. (3) and (4) and may be assumed as

$$F^h(\xi,\eta,t_1) = \bar{N}_\xi \frac{\eta^2}{2} + \bar{N}_\eta \frac{\xi^2}{2} \tag{11}$$

For movable-edge conditions, it can be shown that

$$\bar{N}_\xi = \bar{N}_\eta = 0$$

and therefore

$$F = F^P\, (\xi,\eta,\tau) \tag{12}$$

For immovable-edge conditions, the constants $\bar{N}_\xi$ and $\bar{N}_\eta$ are determined by using the nondimensional form of Eqs. (4)

at $\xi = 0, 1$:

$$F,_{\xi\eta} = 0, \quad \int_0^1 \int_0^1 \left[\beta_1 F,_{\xi\xi} + \gamma\, r^2\, F,_{\eta\eta} - \frac{1}{2} w,_\xi^2\right] d\xi\, d\eta = 0 \tag{13a}$$

at $\eta = 0, 1$:

$$F,_{\xi\eta} = 0, \quad \int_0^1 \int_0^1 [\alpha F,_{\xi\xi} + \gamma^2 \beta_1 F,_{\eta\eta} - (r^2/2)\, w,_\eta^2]\, d\xi\, d\eta = 0 \tag{13b}$$

Substitution of Eq. (11) into Eqs. (13) leads to

$$\bar{N}_\xi = \frac{\alpha I_\xi - \beta_1 I_\eta}{r^2\, (\alpha\gamma - \beta_1^2)} \tag{14a}$$

$$\bar{N}_\eta = \frac{\gamma I_\eta - \beta_1 I_\xi}{\alpha\gamma - \beta_1^2} \tag{14b}$$

where

$$I_\xi = \frac{1}{2} \int_0^1 \int_0^1 w,_\xi^2\, d\xi\, d\eta \tag{15a}$$

$$I_\eta = \frac{1}{2} \int_0^1 \int_0^1 w,_\eta^2\, d\xi\, d\eta \tag{15b}$$

$$\beta_1 = \frac{A_{12}^* \sqrt{D_{11}\, D_{22}}}{h^2} \tag{15c}$$

The particular solution F^p can be determined mathematically by imposing Eq. (9) into Eq. (7). This is shown in Ref. 13.

The total solution for F and the expression for w from Eq. (9) then are substituted into Eq. (6). By making use of the admissible function as a weighted function and setting the weighted residual equal to zero,

$$\int_0^1 \int_0^1 L(w,F)\, \phi\, (\xi,\eta)\, d\xi\, d\eta = 0 \tag{16}$$

the following nonlinear ordinary differential equation is obtained

$$q,_{\tau\tau} + 2\, \xi\omega_0 q,_\tau + \omega_0^2 q + B q^3 = \bar{P}(\tau) \tag{17}$$

where ω_0 and B are defined for both simply supported and clamped plates, and $\bar{P}$ is defined as

$$\bar{P} = \int_0^1 \int_0^1 P_1 \phi\, (\xi,\eta)\, d\xi\, d\eta \tag{18a}$$

and $\bar{P}$ is assumed to be

$$\bar{P} = \bar{P}_0 \cos\omega\tau \tag{18b}$$

As can be seen in Eq. (17), the damping factor also is taken into consideration due to its significant effect on the response of structures. Generally, the damping ratio ξ for composite panels used in aircraft structures ranges from 0.005–0.05.

Application of the Method of Multiple Scales

Before carrying out the analysis using the method of multiple scales, Eq. (17) can be further simplified by using the following transformation

$$t = \omega_0 \tau, \quad q = (h/a)\hat{q} \tag{19}$$

Using this into Eq. (17), we obtain

$$\hat{q},_{tt} + 2\epsilon\mu\hat{q},_t + \hat{q} + \epsilon\hat{q}^3 = F \cos\nu t \tag{20}$$

in which

$$\epsilon = (B/\omega_0^2)(h/a)^2, \quad \mu = \xi/\epsilon, \quad F = (\bar{P}_0/\omega_0^2)(a/h), \quad \nu = \omega/\omega_0 \tag{21}$$

It is to be noted that ϵ is a dimensionless small quantity.

A straightforward-expansion solution of Eq. (20) gives secular terms and small-divisor terms. The small-divisor terms occur when $\nu \approx 1$ (primary resonance) and when $\nu \approx 0$, ⅓, or 3 (secondary resonances). The MMS is a powerful and systematic method in producing an approximate analytical solution to Eq. (20) that is free of secular and small-divisor terms. In the next subsections, we treat the primary and secondary resonances separately by using the method of multiple scales.

Primary Resonance ($\nu \approx 1$)

In order to prevent the small-divisor terms near $\nu \approx 1$ in the straightforward expansion, the forcing term is ordered as $F_0 = \epsilon f$. Equation (20) then is written as

$$\hat{q}_{,tt} + 2\epsilon\mu\hat{q}_{,t} + \hat{q} + \epsilon\hat{q}^3 = \epsilon f \cos\nu t \tag{22}$$

To determine an approximate solution to Eq. (22), the time scales $T_n = \epsilon^n t$ are introduced so that derivatives are transformed as

$$\frac{d}{dt} = D_0 + \epsilon D_1 + \dots \tag{23a}$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\,\epsilon D_0 D_1 + \dots \tag{23b}$$

where $D_n = \partial/\partial T_n$. The following approximate solution to Eq. (22) is sought

$$\hat{q} = q_0(T_0,T_1) + \epsilon q_1(T_0,T_1) + \dots \tag{24}$$

Substituting Eq. (24) into Eq. (22) and equating coefficients of like powers of ϵ, we obtain

$$D_0^2 q_0 + q_0 = 0 \tag{25}$$

$$D_0^2 q_{11} + q_1 = -2D_0D_1q_0 - 2\mu D_0 q_0 - q_0^3 + f\cos\nu T_0 \tag{26}$$

The solution to Eq. (25) may be expressed as

$$q_0 = A(T_1)e^{iT_0} + \bar{A}(T_1)e^{-iT_0} \tag{27}$$

Substitution of Eq. (27) into Eq. (26) gives

$$D_0^2 q_1 + q_1 = -(2iA' + 2i\mu A + 3A^2\bar{A})\,e^{iT_0} - A^3 e^{3iT_0} + ½ f e^{i\nu T_0} + \text{c.c.} \tag{28}$$

where c.c refers to the complex conjugate of the terms on the right side of Eq. (28). The closeness of ν to 1 can be defined by the following equation

$$\nu = 1 + \epsilon\sigma \tag{29a}$$

$$\nu T_0 = T_0 + \sigma T_1 \tag{29b}$$

where σ is a detuning parameter. Substitution of Eq. (29b) into Eq. (28) gives

$$D_0^2\, q_1 + q_1 = -(2iA' + 2i\mu A + 3A^2\bar{A} - ½ f e^{i\sigma T_1})\,e^{iT_0} - A^3 e^{3iT_0} + c.c \tag{30}$$

Elimination of secular terms from Eq. (30) yields the equation governing the complex amplitude of q_0

$$2iA' + 2i\mu A + 3A^2\bar{A} - ½ f e^{i\sigma T_1} = 0 \tag{31}$$

where A is expressed in the polar form as

$$A = ½ a(T_1)\,e^{i\beta(T_1)} \tag{32}$$

Substituting this into Eq. (31) and separating real and imaginary parts yields

$$a' + \mu a - ½ f \sin(\sigma T_1 - \beta) = 0 \tag{33}$$

$$a\beta' - (3/8)\,a^3 + ½ f\cos(\sigma T_1 - \beta) = 0 \tag{34}$$

Next, we let

$$\gamma = \sigma\, T_1 - \beta \tag{35a}$$

and differentiate the result to express β' as

$$\beta' = \sigma - \gamma' \tag{35b}$$

Substitution of Eqs. (35a) and (35b) into Eqs. (33) and (34) gives

$$a' + \mu a - ½ f \sin\gamma = 0 \tag{36}$$

$$a\gamma' - a\sigma + (3/8)\,a^3 - ½ f\cos\gamma = 0 \tag{37}$$

For the steady-state solution $a' = \gamma' = 0$, the following equation can be obtained for the frequency response

$$\mu^2a^2 + a^2\,[\sigma - (3/8)a^2]^2 = 1/4f^2 \tag{38}$$

Once a is known, the solution for $\hat{q}$ is given by

$$\hat{q} = a\cos(\omega t - \gamma) + 0(\epsilon) \tag{39}$$

Secondary Resonances (ν away from 1)

When ν is away from 1, small divisors appear to $0(\epsilon)$ and the forcing function need not be ordered. Again seeking an approximate solution like Eq. (24), one obtains two cases for secondary resonances (details are given in Ref. 13):

1. Subharmonic Resonance, $\nu \approx 3$

In this case, the frequency-response equation is given by

$$9\,\mu^2a^2 + [\sigma - 9H^2 - (9/8)a^2]^2a^2 = (81/16)a^4H^2 \tag{40a}$$

where

$$H = F/2(1-\nu^2) \tag{40b}$$

The solution for $\hat{q}$ is

$$\hat{q} = a\cos\left(\frac{\nu}{3}t - \frac{\nu}{3}\right) + 2H\cos\nu t + 0(\epsilon) \tag{41}$$

2. Superharmonic Resonance, $\nu \approx 1/3$

In this case, the frequency-response equation is given by

$$\mu^2a^2 + [\sigma - 3H^2 - (3/8)a^2]^2a^2 = H^6 \tag{42}$$

and the solution for $\hat{q}$ is

$$\hat{q} = a\cos(3\nu t - \gamma) + 2H\cos\nu t + 0(\epsilon) \tag{43}$$

where

$$\gamma = \sigma T_1 - \beta \tag{44}$$

Results and Discussion

The MMS is used to study the nonlinear damped response of rectangular, specially orthotropic plates subjected to uniform harmonic excitations. The results presented here are for the prebuckling ranges of the in-plane loads given in Ref. 9. The results cover two cases of boundary conditions: simply supported and clamped. Movable and immovable-edge condi-

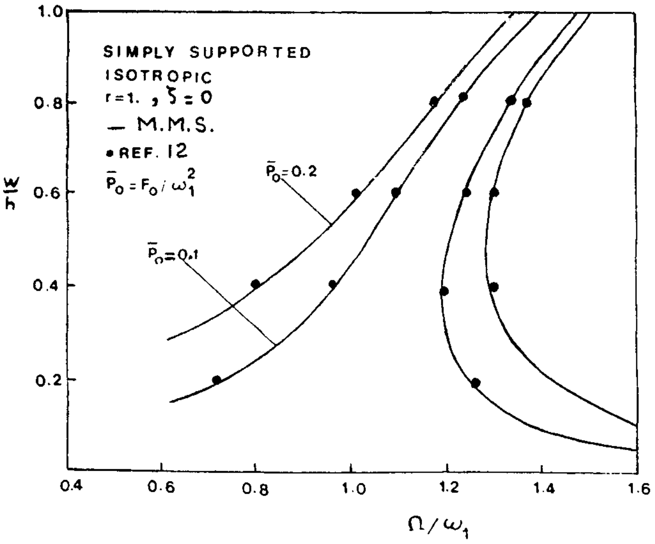

Fig. 1 Comparison of the MMS with FEM for an isotropic plate.

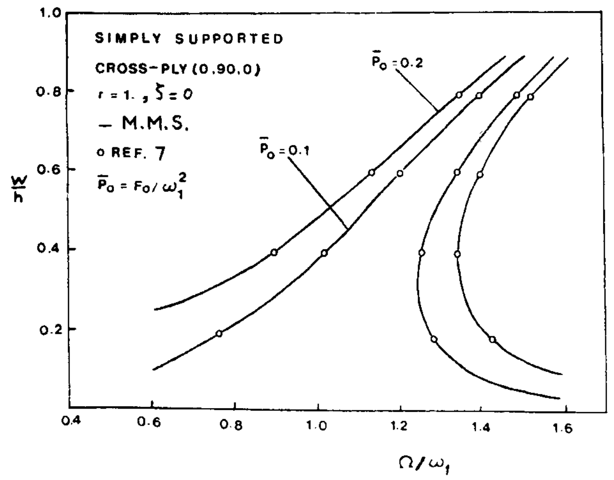

Fig. 2 Comparison of the MMS with FEM for a cross-ply laminated plate.

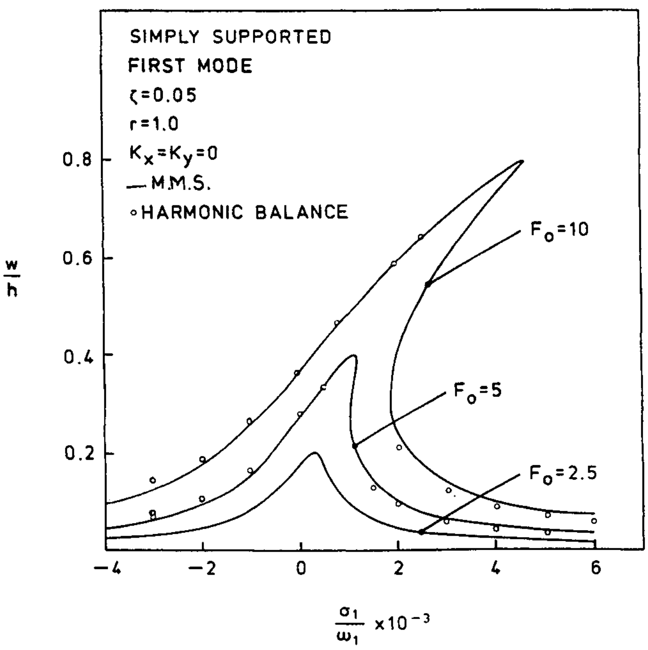

Fig. 3 Effect of the forcing amplitude on primary response and comparison with HBM.

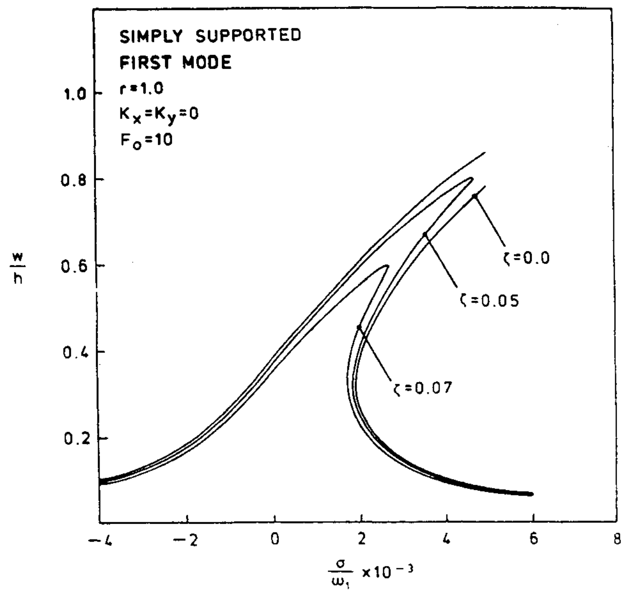

Fig. 4 Effect of the damping on primary response.

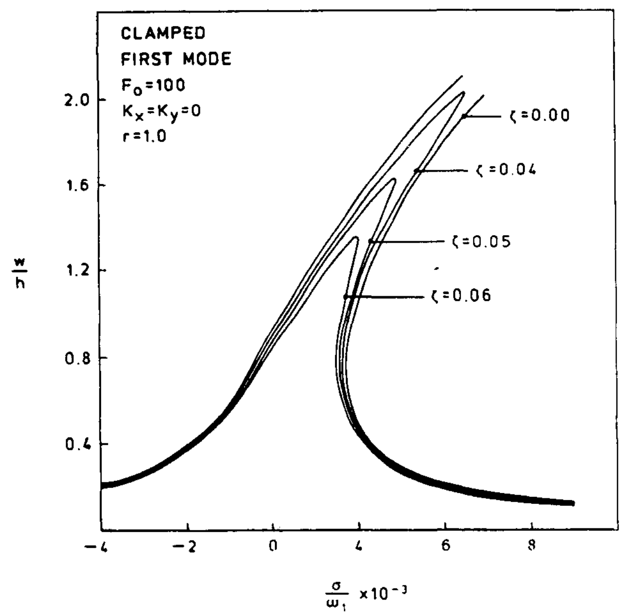

Fig. 5 Effect of the damping on primary response.

tions also are considered. The material considered in the computations of orthotropic plates is Boron-Epoxy with the following properties: $E_1 = 17 \times 10^6$ psi, $E_z = 1.7 \times 10^6$ psi, $G_{12} = 0.68 \times 10^6$ psi, and $\nu_{12} = 0.3$.

Comparisons With the FEM and HBM

The results corresponding to isotropic and Graphite-Epoxy materials used in Refs. 12 and 7 through the finite-element method (FEM) are used here for comparison with the present results (using the same material properties as in Refs. 12 and 7). In Figs. 1 and 2, we show the frequency response curves for a simply supported plate as obtained by the MMS and the comparison with those computed by the FEM.[12,7] These results are for cases without damping or in-plane deformations and inertia. Figure 1 shows the comparison for an isotropic square plate whereas Fig. 2 shows the comparison for a three-layered cross-ply (0 deg, 90 deg, 0 deg) laminated square plate. Obviously, the MMS results are in excellent agreement with the FEM results. Next, we show comparisons of the frequency response curves for an orthotropic plate using the MMS and the harmonic balance method (HBM), Fig. 3. The HBM results also are developed by the authors.

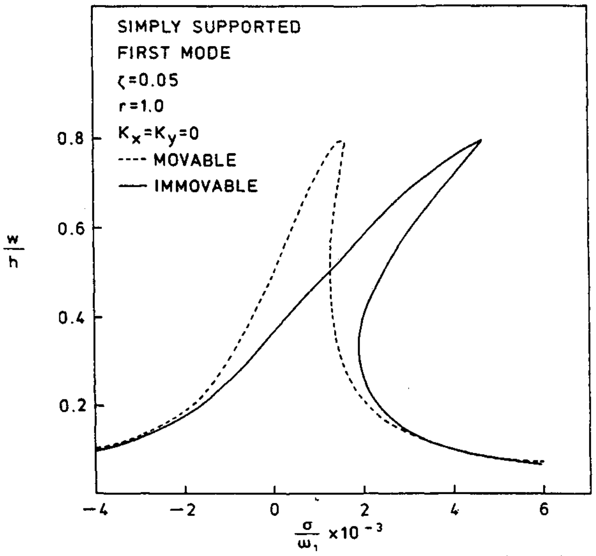

Fig. 6 Comparison of movable and immovable edges for primary response.

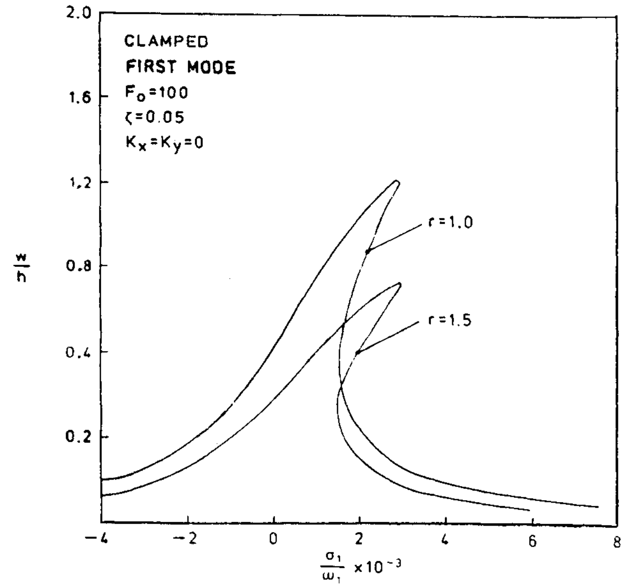

Fig. 9 Effect of aspect ratio on primary response.

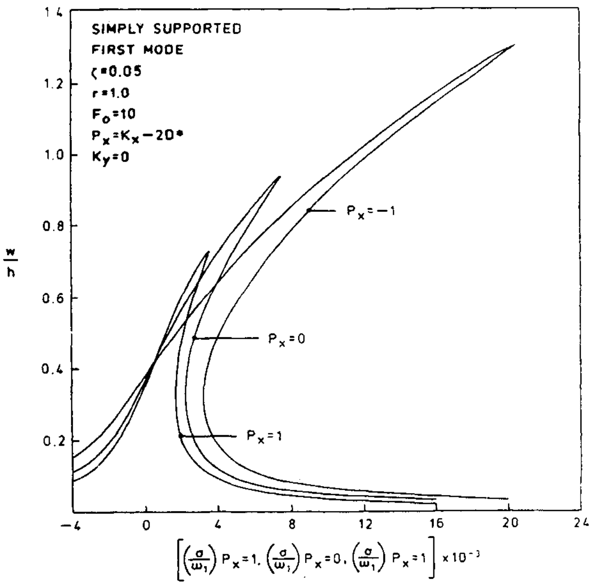

Fig. 7 Effect of in-plane loadings on primary response.

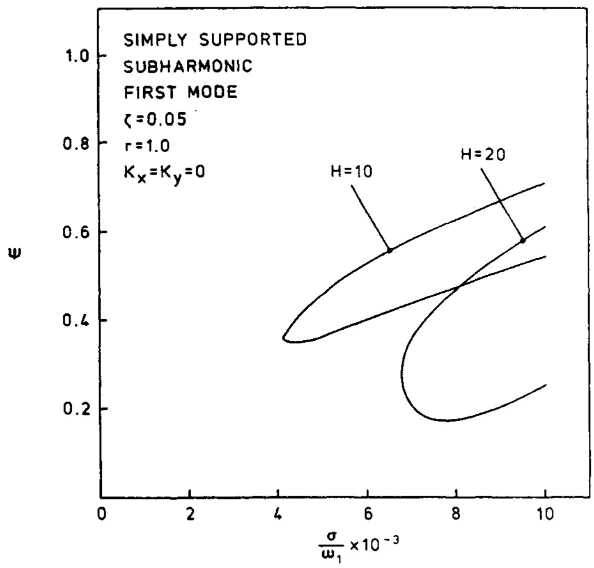

Fig. 10 Typical subharmonic response.

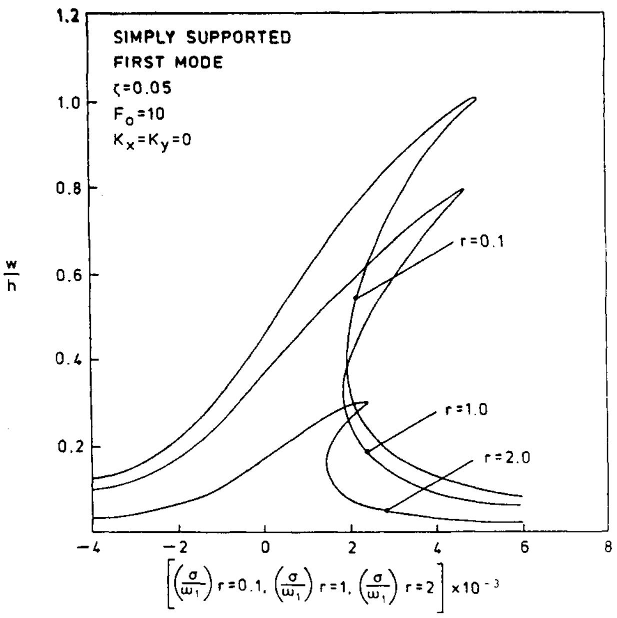

Fig. 8 Effect of aspect ratio on primary response.

Single Mode Response

Using the first mode solution, we show the effects of different parameters on the primary response. Also, we show typical subharmonic and superharmonic responses.

Figures 4 and 5 show the jump phenomena of the primary response for squared simply supported and clamped plates associated with different damping ratios. As the value of damping ratio increases, the peak amplitude decreases.

Figure 6 shows a comparison between the movable and immovable edge conditions for a square simply supported plate associated with a fixed damping ratio. With movable-edge condition, the nonlinearity is significantly decreased.

Figure 7 shows the effect of in-plane loads on the primary-frequency response for a simply supported plate. It is concluded that compressive in-plane loads produce higher amplitudes than those of tensile loads.

Figures 8 and 9 show the effect of aspect ratios on the primary-frequency response for simply supported and clamped plates. For high-aspect-ratio plates, the effect of nonlinearity is stronger than its effect on low-aspect-ratio plates.

Figure 10 and 11 show typical subharmonic responses of simply supported and clamped plates for different values of the forcing parameter. It is to be noticed that although the frequency of the excitation is three times the natural fre-

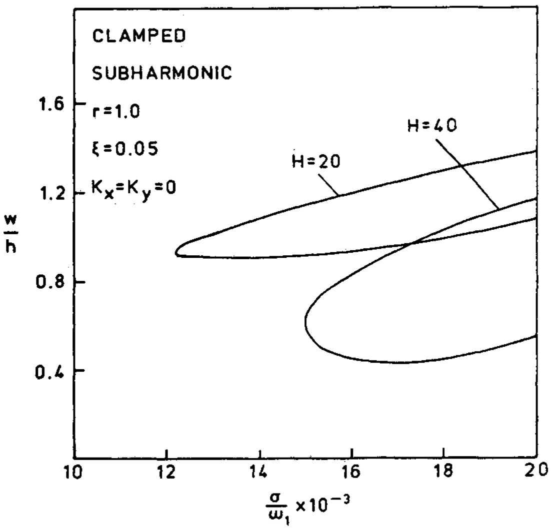

Fig. 11 Typical subharmonic response.

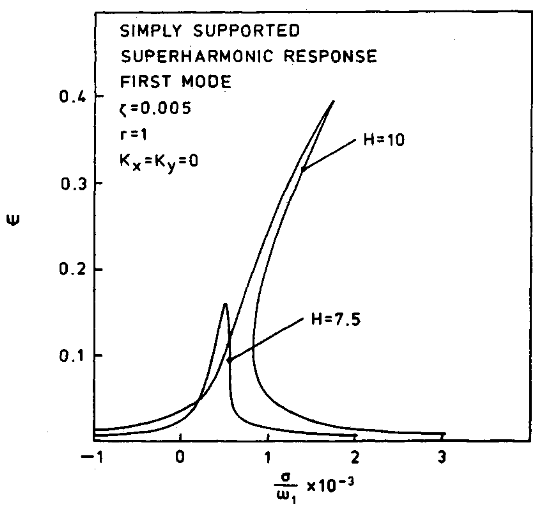

Fig. 12 Typical superharmonic response.

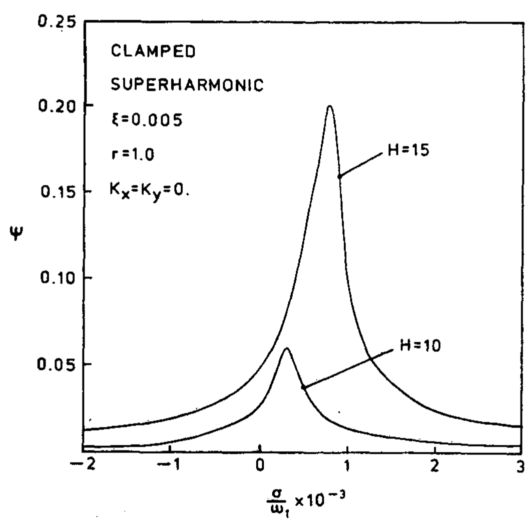

Fig. 13 Typical superharmonic response.

quency, the response is quite large. This gives the motivation to study subharmonic responses. It also is noted that there is no jump phenomenon in this case.

Figures 12 and 13 show typical superharmonic responses of simply supported and clamped plates for different values of the forcing parameter. The jump phenomenon is visible for the simply supported plate with high value of the forcing parameter. For the clamped plate case, the jump phenomenon is not visible.

Concluding Remarks

The MMS is used to study large-amplitude response of specially orthotropic plates subjected to harmonic excitations. The formulation of the problem is based on the von Kármán type geometrical nonlinearity. The effects of in-plane loads is also considered in the formulation. A single-mode Galerkin's method is used to transform the nonlinear coupled partial differential equations into a Duffing-type equation. This equation is solved by the method of multiple scales. Both simply supported and clamped plates with movable and immovable-edge conditions are considered. Comparisons of the present primary response results with those of the existing FEM results and the HBM results show excellent agreement, and they establish confidence and credibility in the present approach. In fact, the present results for the additional cases considered serve as benchmarks for future computational methods. It is concluded that high-aspect ratios, low damping ratios, and high compressive in-plane loads have unfavorable effects.

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